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MODELLING THE RESPONSE OF ISOLATED SLAB SPECIMENS USING EXPLICIT QUASI-STATIC FINITE ELEMENT ANALYSIS

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ABSTRACT

Purpose: This paper assesses the analytic modelling of response of reinforced concrete isolated slab specimens using an explicit quasi-static nonlinear solution rather than the conventional implicit static nonlinear solution. The advantage of the former being its applicability to dynamic problems such as those associated with abnormal loadings on structures.
Design/methodology/approach: Two isolated slab specimens (IA30d-32 and IA30a-25) which have been experimentally tested to fail in flexure and punching shear respectively were adopted for validation of the analytic model.
Findings: Load displacement responses for both implicit static and explicit quasi-static solutions were found to be similar at a loading rate of about $10.575 KNs^{-1}$. Kinetic energy of the system remained zero throughout the duration of the explicit quasi-static simulation with "mean strain rates" ranging between 10^{-8} s^{-1} to 10^{-6} s^{-1}. Hence, the explicit quasi-static solution met the quasi-static loading range specified in Model Code 2010. Results of the analytic assessment were conservative, with 6.4% and 4.1% differences in flexural and punching shear strength respectively and 14% difference in rotation at punching shear failure.
Research limitations/Implications: Assessment was limited to the flexural and punching shear response of isolated flat slab specimens.
Originality/value: It was concluded that at low loading rates, explicit quasi-static analysis could be used in combination with the static failure criterion for the determination of the global response of flat slab structures.

Keywords: Explicit FE analysis; isolated reinforced concrete; flat slabs specimens

1. INTRODUCTION

Buildings are designed to code requirements where partial factors have been adequately incorporated to prevent the attainment of their ultimate limit states. Advances in analytical and computational techniques have led to the development of design codes which are more advanced than those of the past decades. Current design codes have gone beyond the provision of partial factors to only carter for the use of inappropriate material/system models, error in construction and excessive loading. They have also been developed to take into consideration possible occurrence of malevolent, accidental and environmental actions. The latter actions could occur in the form of vehicular, ship or aeroplane impact,

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explosions resulting from gas leaks, terrorist attack or problems in nuclear power plants; or impact and explosion from missiles.

For structural forms such as frame and truss structures, models applied in design codes have adopted various theoretical principles that account for normal stresses, bending, buckling as well as other forms of component responses. Empirical models are applied at local regions which violate the assumptions on which existing theoretical models are based. Slab-column connections of flat slab structures are examples of such local regions.

Flat slabs are reinforced concrete slabs supported directly on columns without beams. Advantages obtained from their incorporation into structures include: ease of construction, reduced storey height and ease of routing of services. Hence, they are significantly applied in the construction of residential and commercial buildings as well as car parking structures. The behaviour of flat slab structures before and after initial local failure is quite different from those of conventional reinforced concrete frame structures. Unlike conventional reinforced concrete framed structures, flat slab structures are not efficient in the transfer of shear forces and moments from the slab to columns through connections. This tends to be significant at edge and corner columns as well as at internal columns with significant differences in lengths of adjacent spans (Sagaseta, Muttoni, Ruiz, and Tassinari, 2011; and Tassinari, 2011).

The design critical mode of failure for flat slab structures is punching shear at the slab-column connection as shown in Figure 1. Punching shear failure is a very brittle form of failure. To improve the strength of slabs as well as ensure a ductile failure mode, punching shear reinforcement is introduced around a control perimeter. Hence, in the design for robustness in flat slab structures, it is very important that the slab-column connections be modelled to effectively cater for possible flexural and punching shear failure modes as well as dynamic response at the connection during the event.

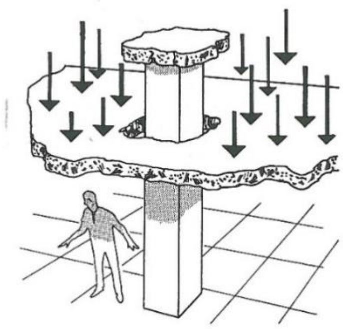

Figure 1a: Falling slab due to punching shear failure at connection

Figure 1b: Punching and post punching shear mechanisms(Source: Mirzael and Muttoni, 2008)

Relationships for punching shear strength of reinforced concrete flat slab connections without shear reinforcement are as shown in equations 1, 2 and 3. Where, V_R is the nominal punching strength, b_0 is the perimeter of the critical section for punching shear, d is the effective depth, f'_c is the specified compressive strength of concrete, ρ is the reinforcement ratio, v_c is the design concrete shear stress, b_v is the breadth of section of slab being considered, A_s is the area of flexural reinforcement provided and ξ is a parameter defined in equation 4. These relationships are as obtained from the American Concrete Institute (ACI) 318 (2011), BS EN 1992-1-1 (2004) and the British Standard (BS) 8110 (1997) respectively. The ACI 318 (2011) expression is empirical and does not take into account the quantity of flexural reinforcement provided at the connection. Hence, it may provide estimates of punching shear strength which may be conservative or unsafe for connections of certain characteristics Muttoni (2008). The BS EN 1992-1-1 (2004) provision for punching shear strength is based on a semi-empirical relationship which directly provides for the influences of the quantity of flexural reinforcement provided.

$$V_R = \frac{1}{3} b_0 d \sqrt{f_c'} \qquad \text{...} Eqn. 1$$

$$V_R = 0.18 b_0 d \xi (100 \rho f_c')^{\frac{1}{3}} \qquad \text{...} Eqn. 2$$

$$v_c = 0.79 \left(\frac{100 A_s}{b_v d}\right)^{\frac{1}{3}} \left(\frac{400}{d}\right)^{\frac{1}{4}} \qquad \text{...} Eqn. 3$$

$$\xi = 1 + \left(\frac{200}{d}\right)^{\frac{1}{2}} \quad (in\ millimeters) \qquad \text{...} Eqn. 4$$

Unlike the empirical and semi-empirical relationships provided in the ACI 318 (2011), BS EN 1992-1-1 (2004) and BS8110 (1997), Muttoni (2008) proposed a physical model capable of accurately predicting not only the punching shear strength of a flat slab connection, but also the slab deformation around the connection. This physical model is termed the Critical Shear Crack Theory (CSCT). The CSCT as presented in Muttoni (2008) is an iterative procedure, where the convergence of a failure criterion and a load rotation curve gives the punching shear strength estimate of the reinforced concrete connection being assessed. Expressions for the failure criterion and load rotation curve are as provided in equations 5 and 6, respectively.

$$\frac{V_R}{b_0 d \sqrt{f_c'}} = \frac{3/4}{1 + 15 \frac{\psi d}{d_{go} + d_g}} \qquad \text{...} Eqn. 5$$

$$\psi = 1.5 \frac{r_s}{d} \frac{f_{yd}}{E_s} \left(\frac{M_{Sd}}{M_{Rd}}\right)^{1.5} \qquad \text{...} Eqn. 6$$

The parameter ψ represents rotation of the slab just outside the column region at a distance of about 2d from the centre of the column (Figure 2). E_s is the modulus of elasticity of flexural reinforcement provided, d_{go} is the reference aggregate size with a value of 16mm, d_g is the maximum aggregate size and r_s is the radius of circular isolated slab element for which a value of 0.22l is adopted for continuous flat slab systems.

Numerical modelling of flat slab-column connections for application to large scale structural systems has posed huge challenges. This is because finite elements (shell elements) which are easily applied to large scale structural systems lack the 3 dimensional stress definitions (Figure 3) necessary for adequate modelling of these connections. However, the adoption of an analytic approach where the load rotation curve of the CSCT is replaced with one obtained through an implicit static non-linear Finite Element Analysis (FEA) (Muttoni *et al.*, 2013; Muttoni and Ruiz, 2012) enables adequate prediction of flexural and punching shear strength of flat slab connections. However, applying this analytic approach to flat slab structural systems under dynamic forms of loading obtainable during progressive collapse will require an explicit dynamic numerical approach through which the punching shear strength of flat slab connections could be easily determined. This would be of significant importance in the robustness assessment of flat slab structures belonging to consequence classes 2b and 3 as specified in the Approved Document A as well as the BS EN 1991-1-7 (2006) part 1-7 , where a direct design approach may be adopted.

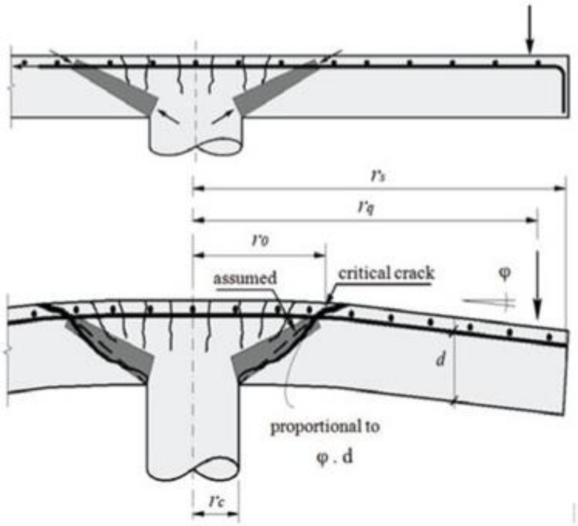

Figure 2: Definitions of parameters of the critical shear crack theory (Sacramento *et al.*, 2012)

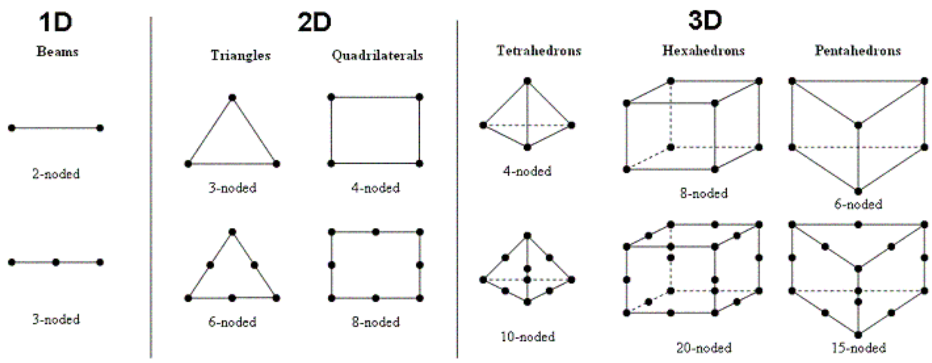

Figure 3: Finite elements

1.1 Objectives of the Study

This study aims at developing an analytic approach for modelling the quasi-static flexural and punching shear response of reinforced concrete isolated slab specimens using an explicit quasi-static finite element analysis with a view to incorporating this analytic approach to slab – column connections in large scale continuous flat slab systems. To achieve the aim the following objectives were developed.

i. Develop an hybrid approach based on an explicit FE analysis and an analytic failure criterion) to assess the response of an isolated slab specimen in flexure and punching shear.
ii. Compare results of the hybrid approach to those obtained through tests and the CSCT for both failure modes.

Finite element modelling will be based on 2 dimensional shell elements with the failure criterion of Muttoni (2008) adopted for the determination of punching strength. The essence of the explicit quasi-static finite element approach is to provide a basis for future application to nonlinear dynamic assessment of flat slab structures subjected to accidental actions.

2. METHODOLOGY

Analytic and numerical methods were applied to this assessment. With validation of results carried out using experimental tests available in literature.

2.1. Experimental tests

Experimental tests carried out on isolated slab specimens by Kinnunen and Nylander (1960) were adopted for the calibration of analytic and numerical models. These tests are as cited in Muttoni (2008) where they were adopted in the comparison of quadrilinear and bilinear moment-curvature relationships applied to load rotation expressions for flat slab connections. Amongst these test specimens presented slab specimens IA30d-32 and IA30a-25 showed distinct differences in their failure modes. They failed in flexure and punching shear respectively, with the percentage of flexural reinforcement they contained being the major difference between both specimens. Recent tests focus on punching shear mode of failure only, making their use for assessment of differing failure models impossible.

Isolated slab specimens are specimens used in experimental testing of flat slab connections. They are developed on the assumption that for a flat slab system, the linear elastic estimate of the line of contra-flexure of radial moments is at a distance $r_s = 0.22L$, from the column centre. Both slab specimens had a slab diameter of 1.84m, column diameter of 0.3m and loadings placed at 12 points along a perimeter with diameter of 1.71m as shown in Figure 4. Material characteristics of slab specimens IA30d-32 and IA30a-25 are as shown in Table 1.

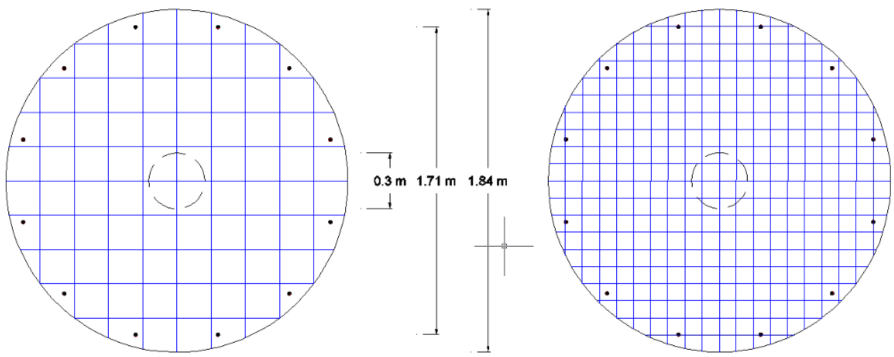

Figure 4: Geometry and reinforcement layout in isolated slab specimens

Table 1: Isolated slab specimen's material Properties

Property	IA30d-32	IA30a-25
Compressive strength of concrete, f_c	26.8 (MPa)	25.6 (MPa)
Yield stress of steel reinforcements, f_y	451 (MPa)	451 (MPa)
Slab effective depth, d	123 (mm)	124 (mm)
Percentage of reinforcement provided, r	0.5 %	1.0 %
Maximum aggregate size, d_g	32 (mm)	32 (mm)

Source: Muttoni (2008); and Kinnunen and Nylander (1960)

2.2. Critical shear crack theory (CSCT)

Using equations 5 and 6 of the CSCT, the load rotation and failure criterion curves were obtained. For each slab specimen, the estimated punching shear strength was obtained through an iterative process where increasing values of rotation at a point 2d from the face of the column was inputted.

2.3. Implicit and explicit finite element modelling

Implicit and explicit finite element modelling was carried out using the commercial finite element code LS-DYNA. Though LS-DYNA also possesses implicit capabilities, the explicit solver was adopted due to the need for the development of an approach which could be applied to the time based assessment of the dynamic response of flat slab systems. To ensure that possible dynamic effects developed during the explicit analyses were negligible, results obtained from the explicit simulations were compared to those obtained from an implicit simulation of the same isolated slab specimen.

Finite element models were developed in accordance with the geometric and support conditions of experimental test. Shell elements were adopted since they were more efficient and were readily applicable to the global assessment of structural systems. Elements were of variable shapes (quadrilaterals and triangles) which allowed for easy compatibility with the circular geometry of the slab and column as shown in Figure 5. Element size was limited to 20mm. Shell elements were defined as layered elements using the keyword *PART_COMPOSITE. The element formulation 2 (Belytschko-Tsay element formulation) was used due to its compatibility with the element type adopted and constitutive material model *MAT_CONCRETE_EC2 to be used. The constitutive material model *MAT_CONCRETE_EC2 allowed for the modelling of the reinforced concrete flat slab sections as consisting of concrete only or concrete and steel reinforcements. A value of 3.5e+9 was used for the tension stiffening parameter "ESOFT". The slab was loaded at 12 points as explained above using a displacement control approach.

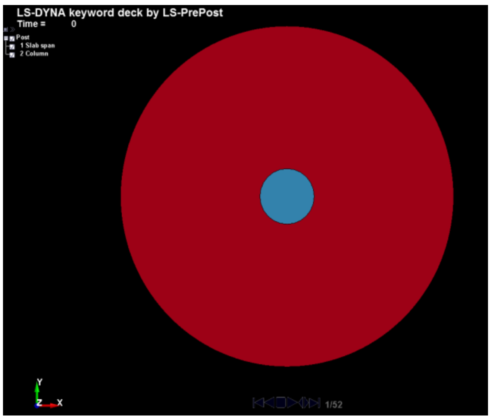

Figure 5a: Finite element model geometry

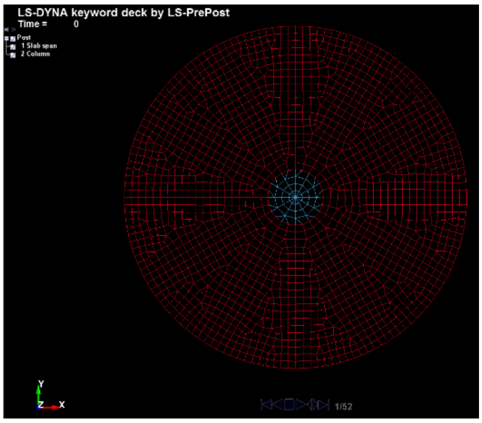

Figure 5b: Finite element mesh

A comparison was made between the implicit and explicit finite element solutions. The objective of this comparison was to ensure that the loading rate adopted for the explicit quasi-static analysis did not allow for the development of dynamic effects. Such dynamic effects could be observed as oscillations in both support reaction and slab displacement responses. A sample loading rate of 10.575KN/s was adopted.

3. Presentation and Discussion of Results

Results of analytic analysis, the numerical modelling as well as their comparison with experimental tests are presented hereunder.

3.1. Experimental tests

Experimental test results were presented as load rotation curves. This allowed for adequate comparison since load and rotation parameters of flat slab connections were the basis of the critical shear crack theory. Tests showed that slab specimen IA30d-32 failed in flexure while slab specimen IA30a-25 failed in punching shear. The only reason for this being the difference in percentages of flexural reinforcement provided. Figure 6 shows the load rotation curves obtained from tests. It is observed that due to very similar concrete properties, slab specimen responses where similar at the pre-cracking phase. However, post-cracking response for slab specimen IA30a-25 was stiffer due to a higher quantity of flexural reinforcement. Slab specimen IA30d-32 showed a very ductile failure relative to that of the specimen IA30a-25 which is very brittle. Though ductility of flat slab connections seems to be a clear distinguishing factor of the prevailing failure mode as observed in Figure 6, this is not the case in all scenario. Provision of an adequate quantity of punching shear reinforcement enhances ductility, though the failure mode still being punching shear failure. Observations of crack patterns developed around the connection remains the ideal approach for the determination of the failure mode.

3.2. Analytic analysis

An analysis of the slab specimens analytically, using the CSCT gave predictions of punching shear strength comparable to those obtained experimentally. The intersection of the parabola (obtained using Equation 6) and the failure criterion curve (obtained using Equation 5) gave the punching shear strength as shown in Figure 6. Flexural strength of Specimen IA30d-32 was obtained through a yield line analysis since equation 5 does not predict the ultimate flexural strength.

3.2.1 Numerical modelling: Comparison between implicit (static) and explicit (quasi-static) solutions

Results of implicit static and explicit quasi-static solutions were quite similar. Explicit solutions gave a more stable solution. The implicit static solution encountered a convergence problem at about 4.5s into the analysis which led to its termination. Load deflection curves obtained through both approaches, prior to the termination time are as shown in Figure 7. Dynamic effects were quite negligible in the explicit quasi-static analysis since its load deflection curve closely matched that of the implicit static analysis.

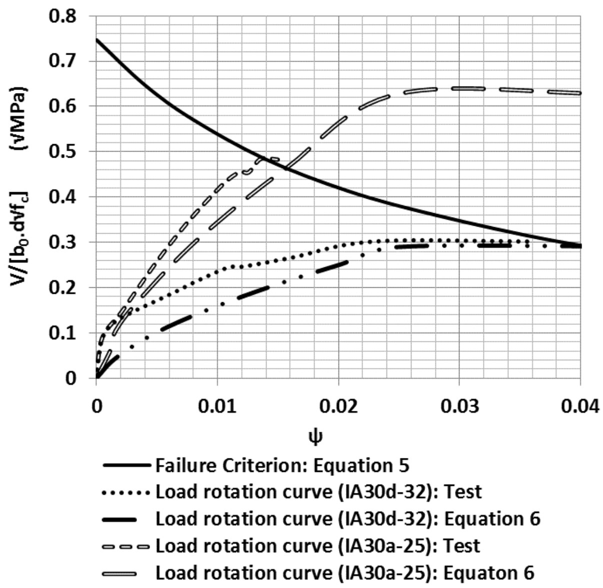

Figure 6: Comparisons of experimental tests and analytic results

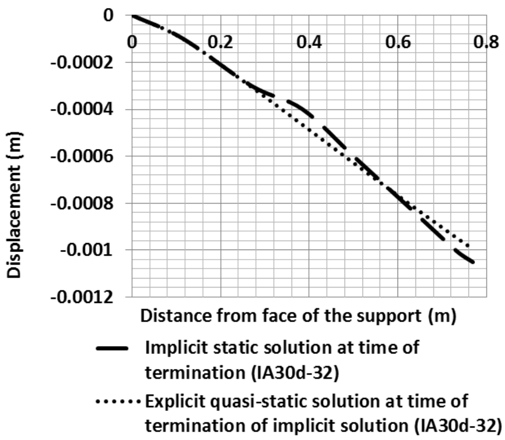

Figure 7: Comparison of implicit and explicit solutions

A check on the kinetic energy of the system gave the occurrence of a zero system kinetic energy through the duration of the simulation. The maximum "mean strain rate" remained within the range of a quasi-static loading as specified in the Model Code 2010. This range was given as 10^{-8} s^{-1} to 10^{-6} s^{-1}.

3.2.2 Explicit quasi-static solution with failure criterion

Punching shear strength of slab specimens were also accurately predicted using load rotation curves obtained numerically through the quasi-static analysis in combination with the failure criterion obtained using Equation 5. A 6.4% difference was obtained between flexural strength obtained numerically to that obtained experimentally for slab specimen IA30d-32. On application of the failure criterion, it was found to intersect the load rotation curve at the plateau as shown in Figure 8. This signified a flexural-punching shear failure as also obtained both experimentally and analytically. Hence, the explicit quasi-static solution with a failure criterion curve accurately predicted response of the slab specimen IA30d-32. The slab specimen was reported to have failed with positive and negative yield lines running radially from the centre. This flexural failure mode was also well predicted by the finite element model as seen in the deformed shape of the slab specimen in Figure 9.

Slab specimen IA30a-25 experimental test results were also well predicted through explicit quasi-static finite element modelling though the post initial cracking stiffness was slightly stiffer than that obtained experimentally. Punching shear strength obtained using the load rotation curve and the failure criterion, was 4.1% lower than that obtained experimentally. Rotation at punching had a slightly higher percentage difference of 14%.

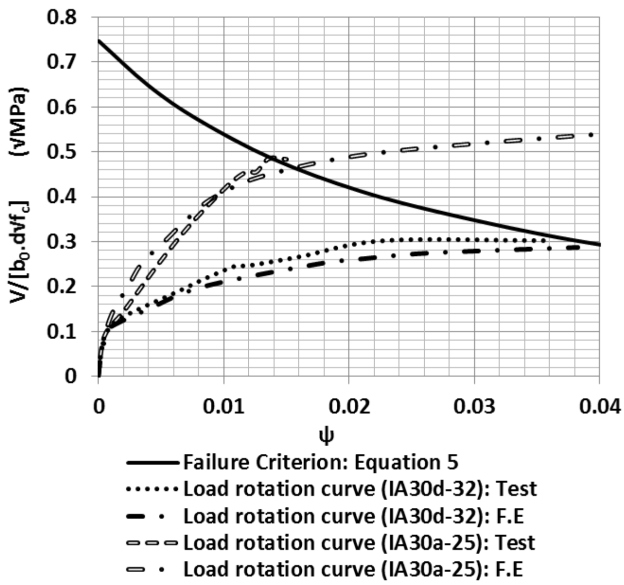

Figure 8: Comparisons of experimental specimen tests and analytic results

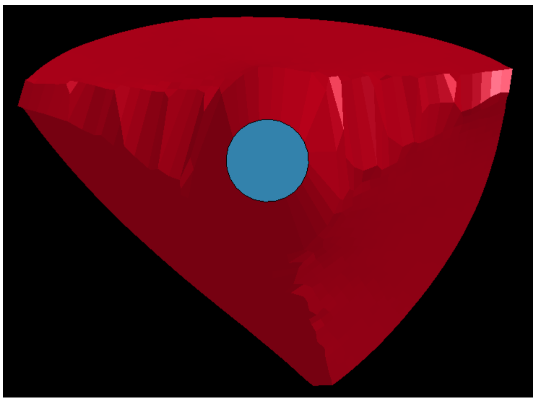

Figure 9: Deformed shape of slab IA30d-32 at failure

4. Conclusion and Recommendations

The design of new structures using modern codes of practice makes great emphasis on the robustness of these structures. It is expected that in the event of accidental or malicious events which may subject such structures to abnormal loading scenarios, the structure will not be damaged to an extent disproportionate to the cause. The peculiarity of flat slab structures, notably the possible occurrence of punching shear failure at the slab-column connections, poses some challenges in efficient modelling of such structures. Adoption of a static nonlinear finite element model of the slab flexural response in addition to the failure criteria of the Critical Shear Crack theory allows for the effective prediction of the static response of flat slab connections and hence the global structure.

However, progressive collapse being a dynamic event necessitates the development of an explicit approach through which the flexural response of the slab could be obtained under the quasi-static and dynamic situations obtainable when modelling structures for progressive collapse events. On application of the explicit quasi-static approach to two isolated flat slab specimens, results obtained were comparable to those obtained experimentally and theoretically, both for the flexural and punching shear failure modes. The study therefore concludes that at low loading rates an explicit quasi-static analysis could be used in combination with the static failure criterion for the determination of the local punching shear failure in flat slab structures.

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